Some Results about the Distinction of Side-Channel Distinguishers based on Distributions

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Presentation Outline

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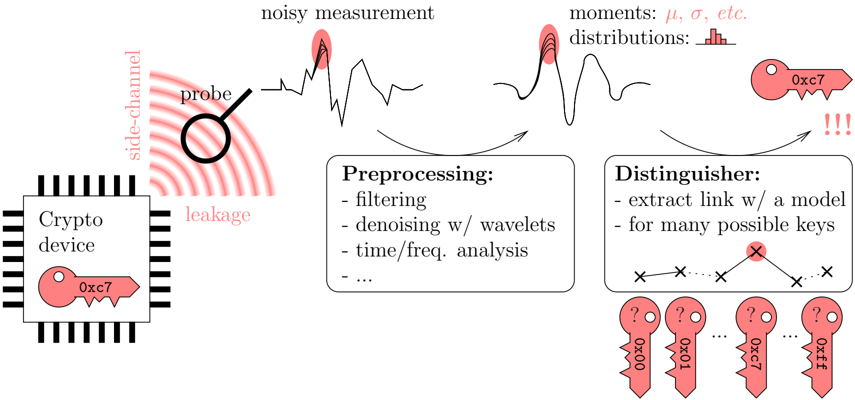
Side Channel Analysis



Classical distinguishers

- **Covariance** used for Differential Power Analysis (Kocher *et al.* [5])
- **Correlation** calculated with the Pearson's coefficient (Brier *et al.* [1])
- **Likelihood**, as in Template Attacks (Chari *et al.* [2])
- **Mutual Information** Analysis (Gierlichs *et al.* [4])

Information theoretic definitions

Notation : Let $p(X = x)$ **be abridged** $p(x)$

- Mutual information, a general measure of dependence:

$$II(X;Y) = H(X) - H(X \mid Y) \geq 0$$

- Shannon's entropy, a measure of information:

$$H(X) = -\sum_{i=1}^{n} p(x_i) \cdot \log p(x_i)$$

- Conditional entropy:

$$H(X \mid Y) = -\sum_{i=1}^{n} \sum_{j=0}^{m} p(x_i, y_j) \cdot \log p(x_i | y_j)$$

Probability density function estimation

- Non-parametric methods:
  - Histogram
  - Kernel density:

$$f(x) = \frac{1}{nh} \sum_{i=0}^{n} k\left(\frac{x - x_i}{h}\right),$$

where k is the kernel function and h is the "bandwidth"

- Parametric method:

$$f(x) = \sum_{i=0}^{n-1} w_i \cdot \mathcal{N}(x, \mu_i, \sigma_i),$$

where w_i, μ_i and σ_i are the weight, the mean and the standard deviations, respectively.

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Conventional metrics

- The metric is an average distance $\mathcal{D}$ over Y between $p(x|Y)$ and its average $\mathbb{E}(p(x|Y)) = p(x)$:

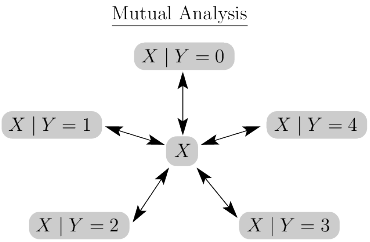
$$M_0 = \mathbb{E}\big(\mathcal{D}(p(x|Y), p(x))\big) = \sum_y p(y)\mathcal{D}(p(x|y), p(x))$$

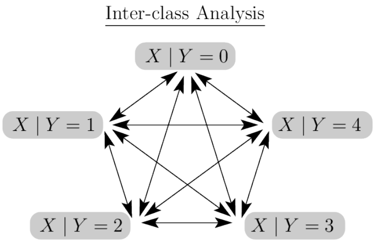
- Examples of distances $\mathcal{D}$ are:
  - MIA : Kullback-Leibler divergence
  - Total variation $D(p,q) = \sum |p - q|$
  - KSA : Kolmogorov-Smirnov distance (cumulative distribution functions).

Inter-class metric

The inter-class is a distance $\mathcal{D}$ between two classes $p(x|Y)$ and $p(x|Y')$

$$M_I = \frac{1}{2}\mathbb{E}(\mathcal{D}(p(x,Y),p(x,Y'))) = \frac{1}{2}\sum_{y,y'} p(y)p(y')\mathcal{D}(p(x|y),p(x|y'))$$





Contributions

1. Inter-class Information Analysis (IIA), a new test for SCA proved to be efficient when exploiting several kinds of leakages
2. A theoretical study to compare the inter-class information with the mutual information
3. Some experiments in order to evaluate IIA (simulated attacks against unprotected and protected AES with Boolean masking)

First definition

$$II(X;Y) = \frac{1}{2}\mathbb{E}(D_{\mathrm{KL}}[X \mid Y \parallel X \mid Y'])$$

Where $D_{\mathrm{KL}}[X \parallel Y]$ is the Kullback Leibler divergence

Second definition

- $$II(X;Y) = \frac{H'(X \mid Y) - H(X \mid Y)}{2}$$
- Recall the conditional entropy is averaged over the joint distribution: $H(X \mid Y) = \sum_{x,y} p(x,y) \log \frac{1}{p(x|y)}$
- The "cross-entropy" is averaged over the product distribution: $H'(X \mid Y) = \sum_{x,y} p(x)p(y) \log \frac{1}{p(x|y)}$

Properties

- **Relation to Mutual Information** Since $2II(X;Y) = \mathbb{E}(D_{\mathrm{KL}}[X \mid Y \parallel X] + D_{\mathrm{KL}}[X \parallel X \mid Y]) = I(X;Y) + \mathbb{E}(D_{\mathrm{KL}}[X \parallel X \mid Y])$

$$\boxed{2II(X;Y) \geq I(X;Y)}$$

- **Symmetry** Since $2II(X;Y) = I(X;Y) + \sum_{x,y} p(X=x)p(Y=y) \log \frac{p(X=x)p(Y=y)}{p(X=x,Y=y)}$

$$\boxed{II(X;Y) = II(Y;X)}\ .$$

- **Independence** $II(X;Y) = 0$ implies $I(X;Y) = 0$ (independence) Conversely, if X and Y are independent, $D_{\mathrm{KL}}[X \parallel X \mid Y] = D_{\mathrm{KL}}[X \mid Y \parallel X] = D_{\mathrm{KL}}[X \parallel X] = 0$, hence $II(X;Y) = 0$ So:

$$\boxed{II(X;Y) = 0 \iff X,\ Y \text{ are independent}}\ .$$

Soundness Proof

Let k^* the good key and k another key $k \neq k^*$,,
The leakage X is continuous, the model Y is discrete. The PDF of $X = Y(k^*) + N = \phi(Z, k^*) + N$ (where N is an arbitrary RV modelling the noise)
For each y, $X|y$ is a PDF mixture whose coefficients depend on y, and whose modes y^* take values on a finite set

- **MI**: Moradi et al [6] relies on the fact that $Y \to Y^* \to X$ (where $k \neq k^*$) forms a Markov chain; then by the well-known information inequality for Markov chains, $I(X; Y^*) \geq I(X; Y)$.
- **II**: Since $H(X|Y) > H(N) = H(X|Y^*)$, we end up with

$$II(X;Y) = \frac{H'(X \mid Y) - H(X|Y)}{2} < \frac{H'(X \mid Y) - H(X|Y^*)}{2}$$
$$< \frac{H'(X \mid Y^*) - H(X|Y^*)}{2} = II(X;Y^*)$$

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IIA and MIA under the Gaussian mixture

When using the good key and for a Gaussian noise N, we have:

$$II(X; Y^*) = \frac{H'(X \mid Y^*) - H(N)}{2} = \frac{\log e}{2} \cdot \frac{\sigma^2_{Y^*}}{\sigma^2} \quad ,$$

$$I(X; Y^*) = \frac{H(Y^* + N) - H(N)}{2} = \frac{1}{2} \log \frac{\sigma^2_{Y^*} + \sigma^2}{\sigma^2} \quad .$$

Key Inequality

$$I(X; Y^*) = \frac{1}{2} \log \frac{\sigma^2_{Y^*} + \sigma^2}{\sigma^2} \leq \frac{\log e}{2} \frac{\sigma^2_{Y^*}}{\sigma^2} = II(X; Y^*)$$

The difference $II(X; Y^*) - I(X; Y^*)$ increases with the SNR $\frac{\sigma^2_{Y^*}}{\sigma^2}$.

- With the help of the Markov chain condition on $Y \to Y^* \to X$, we have:

$$\begin{cases} I(X;Y) \leq I(Y;Y^*) \ll I(X;Y^*) \\ II(X;Y) \leq II(Y;Y^*) \ll II(X;Y^*) \end{cases}$$

- Noting the nearest-rival distinguishability by ζ:

$$\begin{cases} \zeta_{\mathrm{MIA}}(k) = I(X;Y(k^*)) - \max_{k \neq k^*} I(X;Y(k)) \ , \\ \zeta_{\mathrm{IIA}}(k) = II(X;Y(k^*)) - \max_{k \neq k^*} II(X;Y(k)) \end{cases}$$

- By neglecting the second-order terms, we end up with:

$$\boxed{\zeta_{\mathrm{IIA}}(k) \gtrsim \zeta_{\mathrm{MIA}}(k)}$$

- For low SNR, the equality holds at the limit: both methods coincide.

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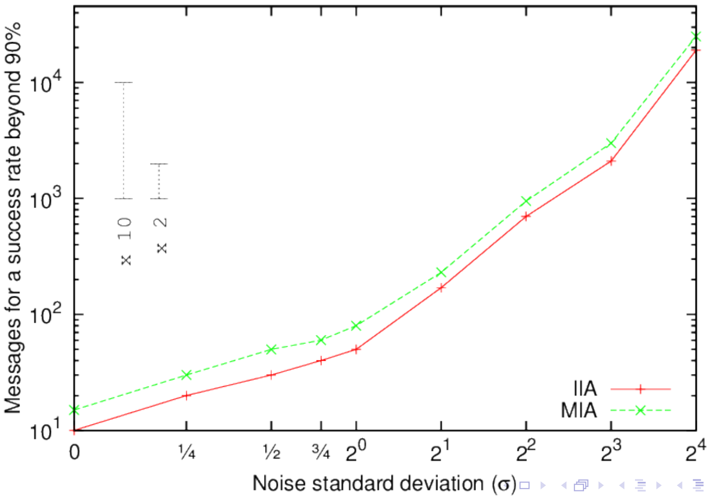
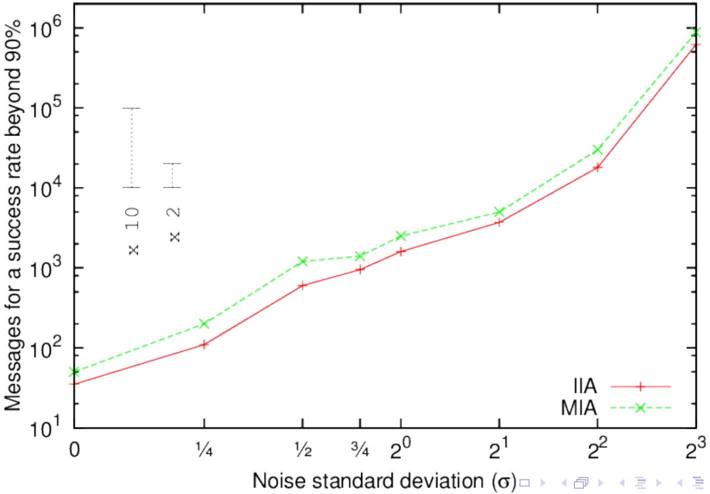
Evaluation Methodology

- *Leakage model*: Hamming distance
- *Target leakage*:
  - 1^{st}-order leakage of unprotected implementation: $X = f(Y) + N = HW(Y) + N$;
  - 2^{nd}-order leakage of 1^{st}-order Boolean masking scheme (Standaert *et al.* [7]): $X = f(Y, M) + N = HW(Y \oplus M) + HW(M) + N$
- *Target secret*: AES S-box output, $Y = S(Z \oplus k)$
- *Adversary model*: known plaintext model by estimating $I(X; \psi(Y))$ and $II(X; \psi(Y))$
- *Prediction function*: Hamming Distance
- *PDF estimation method*: histogram-based PDF estimation

Unprotected implementation



Masked implementation



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Conclusions

- New "Inter-class" concept to distinguish between various partitionings
- Mathematical analysis of the II metric and comparison with the MI metric
- Simulations confirm that IIA is more efficient than MIA in discriminating correct key assumptions:
  - For the usual HD leakage models
  - For unprotected and masked implementation

Perspectives

- Compare this distinguisher with an Inter-class KS analysis (IKSA)
- Try other kinds of masking (*e.g.*, multiplicative or affine masking (Fumaroli et al [3])

Thank you for your attention.

Any Questions?

References

[1] Éric Brier, Christophe Clavier, and Francis Olivier.
Correlation Power Analysis with a Leakage Model.
In *CHES*, volume 3156 of *LNCS*, pages 16–29. Springer, August 11–13 2004.
Cambridge, MA, USA.

[2] Suresh Chari, Josyula R. Rao, and Pankaj Rohatgi.
Template Attacks.
In *CHES*, volume 2523 of *LNCS*, pages 13–28. Springer, August 2002.
San Francisco Bay (Redwood City), USA.

[3] Guillaume Fumaroli, Ange Martinelli, Emmanuel Prouff, and Matthieu Rivain.
Affine Masking against Higher-Order Side Channel Analysis.
In Alex Biryukov, Guang Gong, and Douglas R. Stinson, editors, *Selected Areas in Cryptography*, volume 6544 of *LNCS*, pages 262–280. Springer, 2010.

[4] Benedikt Gierlichs, Lejla Batina, Pim Tuyls, and Bart Preneel.
Mutual information analysis.
In *CHES, 10th International Workshop*, volume 5154 of *Lecture Notes in Computer Science*, pages 426–442. Springer, August 10-13 2008.
Washington, D.C., USA.

[5] Paul C. Kocher, Joshua Jaffe, and Benjamin Jun.
Differential Power Analysis.
In *CRYPTO*, volume 1666 of *LNCS*, pages pp 388–397. Springer, 1999.

[6] Amir Moradi, Nima Mousavi, Christof Paar, and Mahmoud Salmasizadeh.
A Comparative Study of Mutual Information Analysis under a Gaussian Assumption.
In *WISA*, volume 5932 of *LNCS*, pages 193–205. Springer, August 25-27 2009.
Busan, Korea.

[7] François-Xavier Standaert, Gaël Rouvroy, and Jean-Jacques Quisquater.

FPGA Implementations of the DES and Triple-DES Masked Against Power Analysis Attacks.
In *FPL*. IEEE, August 2006.
Madrid, Spain.