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Optimal CPA Attack

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Outline of the talk

- Motivation
- Relation between CPA and the stochastic approach
- The optimal CPA function
- Experimental results
- Conclusion

Motivation

- Dozens of papers on CPA attacks have been published. Typically, CPA attacks focus on a single bit or apply a Hamming weight (distance) model.
- Natural questions:

  For a given implementation:
  - What is the most efficient CPA function?
  - What is the (theoretical) limit of a univariate CPA attack?

What's new?

- This talk considers univariate CPA attacks on unprotected block ciphers.
- We develop an explicit expression for the optimal CPA function (in dependency of the leakage distribution) for leakage models with given complexity.

Reference:

[1] Michael Kasper, Werner Schindler: Towards Optimal Correlation Analysis Attacks by High-Dimensional Stochastic Models. Status: Submitted.

Definition

$x \in \{0,1\}^p$ set of admissible (known) parts of the plaintext or ciphertext
[AES: typically, p = 8 or p = 16]

$k' \in \{0,1\}^s$ set of admissible subkeys
[AES: typically, s = 8]

k correct subkey

$f_t : \{0,1\}^p \times \{0,1\}^s \rightarrow R$ CPA function at time t;
[e.g., Hamming weight of an Sbox output]

$$\rho_N(i_t, f_t, k') = \frac{\frac{1}{N}\sum_j i_t(x_j, k) f_t(x_j, k') - mean(i_t)mean(f_t)}{\sqrt{empVar(i_t)}\sqrt{empVar(f_t)}}$$

CPA

- We interpret $x_1, x_2, \ldots, x_N$ as realizations of iid (independent and identically distributed) random variables $X_1, X_2, \ldots, X_N$.
- As N tends to infinity the term $\rho_N(i_t, f_t, k')$ converges to the correlation coefficient

$$corr(I_t(X,k), f_t(X,k')) = \frac{cov(I_t(X,k), f_t(X,k'))}{\sqrt{Var(I_t(X,k))}\sqrt{Var_X(f_t(X,k'))}}$$

- The CPA function $f_{t,2}$ is viewed better than $f_{t,1}$, if the absolute value $|corr\left(I_t(X,k), f_{t,2}(X,k)\right)|$ is larger than $|corr\left(I_t(X,k), f_{t,1}(X,k)\right)|$.

The stochastic model

$$I_t(x,k) = h_t(x,k) + R_t$$

Random variable
(depends on x and k)

quantifies the random-ness of the side-channel signal at time t

deterministic part
= leakage function
(depends on x and k)

Random variable

$E(R_t) = 0$

Noise (centered)

Stochastic approach: Profiling, Step 1

- For fixed subkey $k \in \{0,1\}^s$ the unknown function

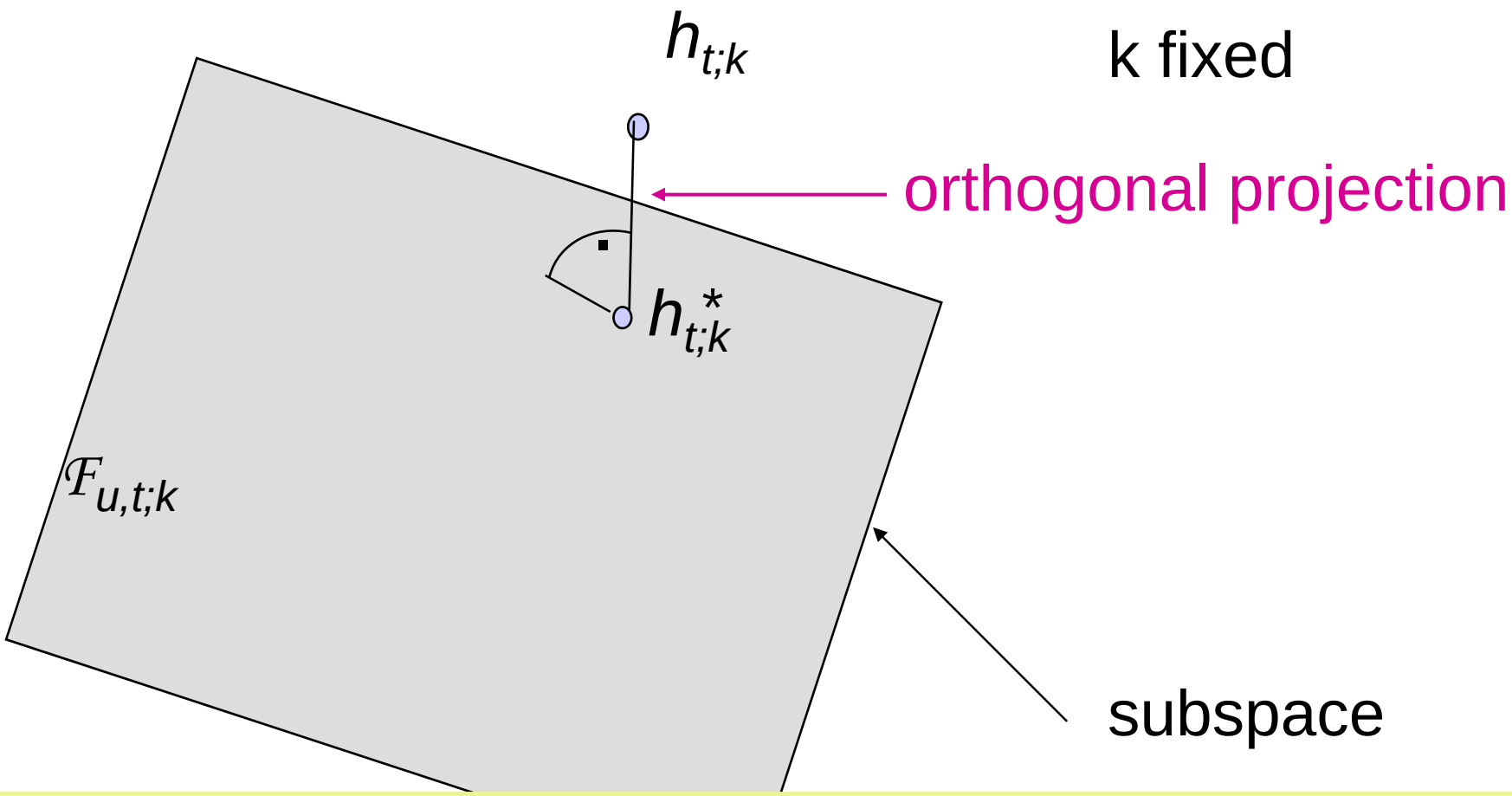
$$h_{t;k}: \in \{0,1\}^p \times \{k\} \to R, \quad h_{t;k}(x;k) := h_t(x;k)$$

is interpreted as an element of the real vector space

$$\mathcal{F}_k := \{h': \{0,1\}^p \times \{k\} \to R\} \qquad [\dim(\mathcal{F}_k) = 2^p]$$

- Stochastic approach: Approximate the leakage function $h_{t;k}$ by its image $h^*_{t;k}$ under the orthogonal projection onto a suitably selected low-dimensional vector subspace $\mathcal{F}_{u,t;k}$.

Geometric illustration



The image $h^*_{t,k}$ is the best approximator of $h_{t;k}$ in $\mathcal{F}_{u,t;k}$

A scalar product on $\mathcal{F}_k$

- Assume that Prob(X=x) > 0 for all $\{0,1\}^p$. (Otherwise cancel those elements that occur with probability 0.)
- Then

  $(f_1,f_2) := \sum_{x\in\{0,1\}^p} Prob(X=x) f_1(x,k) f_2(x,k) = E(f_1 f_2)$

  defines a scalar product on $\mathcal{F}_k$
- Here and in the following $g_{0,t;k} = 1$, $g_{1,t;k},\ldots,g_{2^p-1,t;k}$ denotes an orthonormal basis of $\mathcal{F}_k$ w.r.t $(\cdot,\cdot)$.
- In particular,

$$(h_{t;k}, g_{i,t;k}) = \left(\sum_{j=0}^{2^p-1} \beta_{j,t;k} g_{j,t;k}, g_{i,t;k}\right) = \beta_{i,t;k}$$

CPA (from a more theoretical standpoint)

- Assume that $E(f_t(X,k)) = 0$. Then

$$\mathrm{corr}(l_t(X,k),f_t(X,k)) = \ldots = \mathrm{const}\cdot(h_{t;k}(X,k), \frac{f_t(X,k)}{\|f_t(X,k)\|}),$$

- CPA with the function $f_t(x,k)$ basically corresponds to the stochastic approach with the 1-dimensional subspace $< f_t(\cdot,k) >$

Optimal CPA

Theorem: Assume that $g_{0,t;k} = 1, g_{1,t;k}, \ldots, g_{u-1,t;k}$ is an orthonormal basis of $\mathcal{F}_{u,t;k}$, and $\sigma^2 := \mathrm{Var}(R_t)$.

Then $f_{opt,t} := h_{t;k}^* = \sum_{j=0}^{u-1} \beta_{j,t;k}\, g_{j,t;k}$ is the optimal CPA function, which is contained in the subspace $\mathcal{F}_{u,t;k}$.

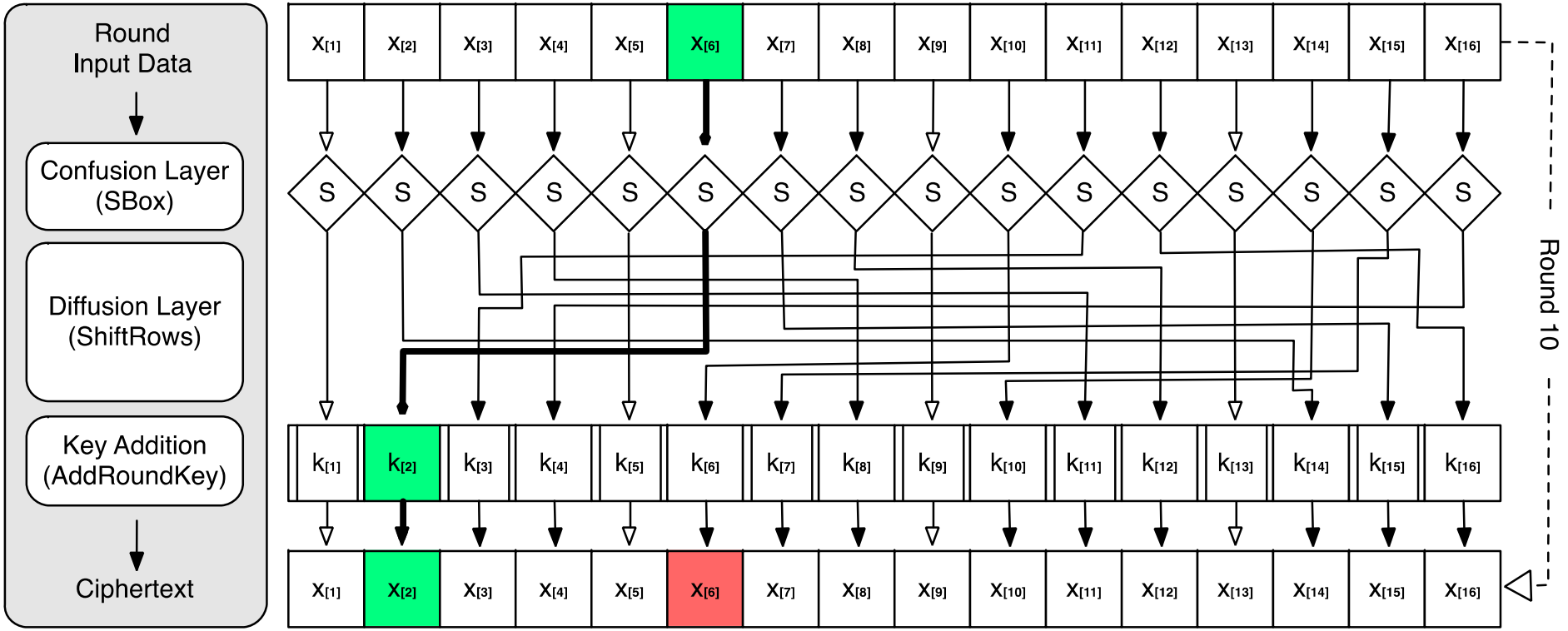
In particular,

$$\mathrm{corr}(I_t(X,k), f_{opt,t}(X,k)) = \frac{\sqrt{\sum_{j=1}^{u-1} \beta_{j,t;k}^2}}{\sqrt{\sum_{j=1}^{2\hat{}p-1} \beta_{j,t;k}^2 + \sigma^2}} = \frac{\sqrt{Var_X(h_{t;k}^*(X,k))}}{\sqrt{Var(I_t(X,k))}}$$

Practical relevance?

- We have determined an explicit formula for the optimal CPA function in different subspaces.
- How large is the information gain compared to ‘standard’ CPA functions?

AES: Final round



Leakage model

9-dimensional subspace

$g_{0,t;k}(x,k) := 1$

$g_{j,t;k}(x,k) := (x_{[b]} \oplus S^{-1}(x_{[a]} \oplus k_{[a]}))_j - 0.5$

for $1 \leq j \leq 8$

Here $x = (x_{[a]},x_{[b]})$ and $k = k_{[a]}$, e.g. (a,b) = (2,6).
Moreover,

$\hat{g}_{j,t,k}(x,k) := gj_{,t,k}(x,k) + 0.5$ for $1 \leq j \leq 8$

$$\mathcal{B}_i := \{\hat{g}_{j_1,t;k} \cdots \hat{g}_{j_i,t;k} - 2^{-i} \mid 1 \leq j_1 < \ldots < j_i \leq 8\}$$

$$\mathcal{B}_{1/2} := \{g_{1,t;k} + \cdots + g_{8,t;k}\}$$

High dimensional leakage models

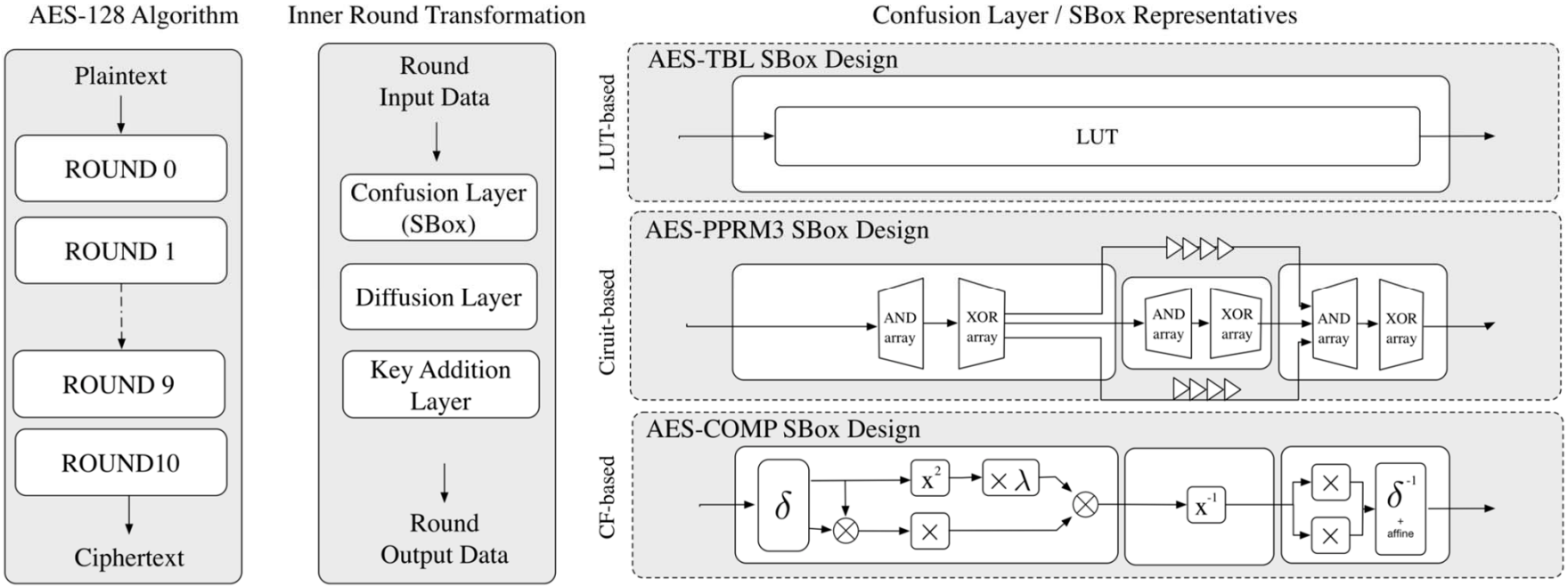
$\dim(\mathcal{F}_{u,t;k}) = u$	Set of basis functions
2	$\mathcal{B}_0 \cup \mathcal{B}_{1/2}$
9	$\mathcal{B}_0 \cup \mathcal{B}_1$
37	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2$
93	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3$
163	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 \cup \mathcal{B}_4$
219	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 \cup \mathcal{B}_4 \cup \mathcal{B}_5$
247	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 \cup \mathcal{B}_4 \cup \mathcal{B}_5 \cup \mathcal{B}_6$
255	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 \cup \mathcal{B}_4 \cup \mathcal{B}_5 \cup \mathcal{B}_6 \cup \mathcal{B}_7$
256	$\mathcal{B}_0 \cup \mathcal{B}_1 \cup \mathcal{B}_2 \cup \mathcal{B}_3 \cup \mathcal{B}_4 \cup \mathcal{B}_5 \cup \mathcal{B}_6 \cup \mathcal{B}_7 \cup \mathcal{B}_8$

High-dimensional leakage models also capture interactions between bit lines, in particular propagation glitches or cross-talk phenomena.

AES Sbox Design

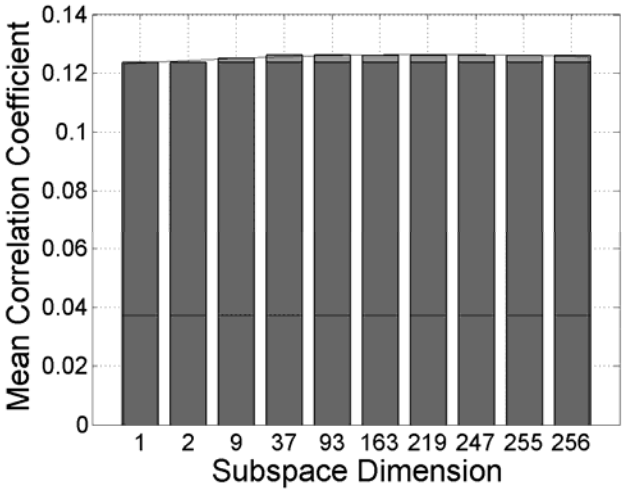
The SBox design affects the power consumption considerably. In our experiments we considered three different design principles:

- AES-TBL: Look-up table-based SBox Design (LUT-based)
- AES-PPRM3: Circuit-based SBox Design
- AES-COMP: Composite-field-based SBox Design

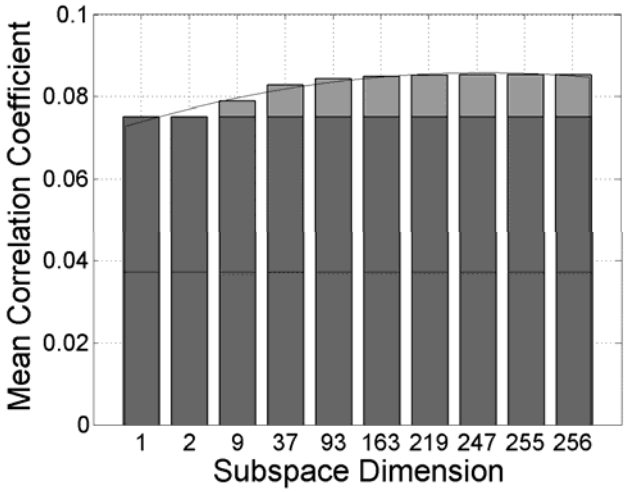


Correlation coefficients (averaged over all key bytes)

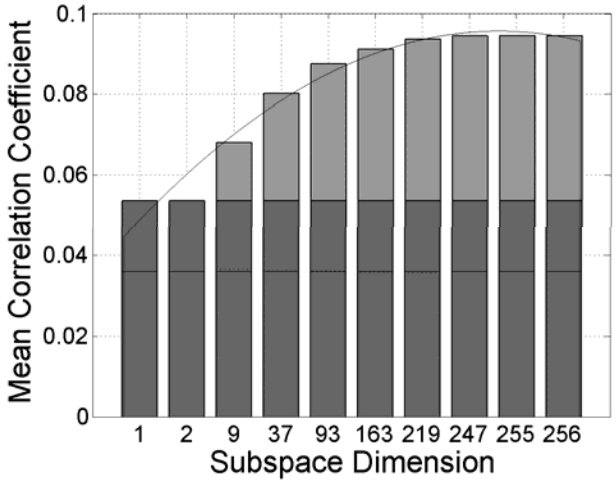
AES-TBL



AES-PPRM3



AES-COMP



1 : ‘Standard CPA’ (Hamming distance model)

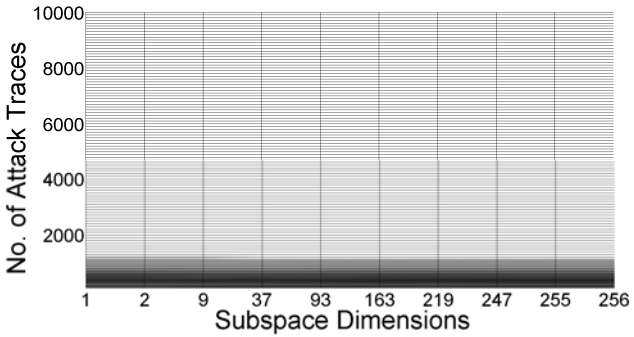
2 – 256: Optimal CPA (high-dimensional stochastic models; u = 2, …, 256)

Information gain

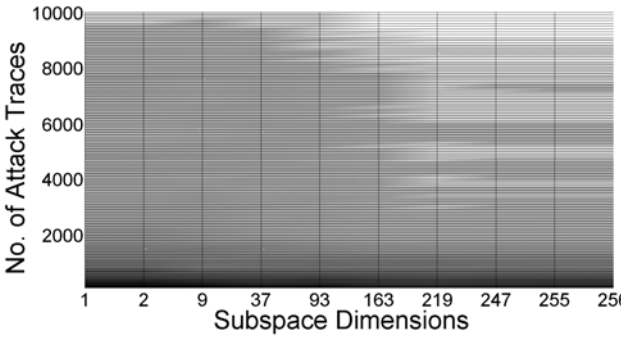
- The ratios of the averaged correlation coefficients (optimal CPA to ‘standard’ CPA) are 1.0190, 1.1350, and 1.7634 (for AES-TBL, AES-PPRM3, AES-COMP)
- *For AES-COMP the **information*** **gain** of the optimal CPA compared to ‘standard’ CPA (HD model) is maximal.
- For AES-COMP the high-dimensional subspaces $\mathcal{F}_{u,t;k}$ capture the leakage much better than the standard CPA.

Global success rate

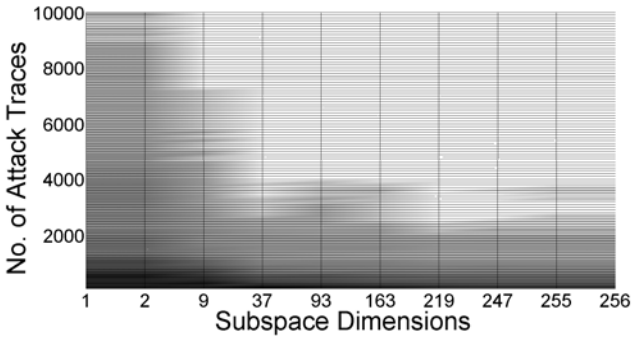
AES-TBL



AES-PPRM3



AES-COMP

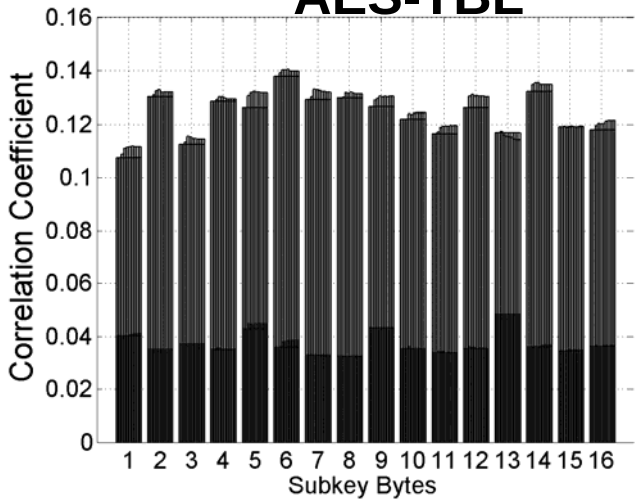


White colouring means ‘attack successful’.

- For AES-COMP the minimum number of power traces needed for stable GSR decreases from 10.000 (standard CPA) to 2.300 traces (optimal CPA).

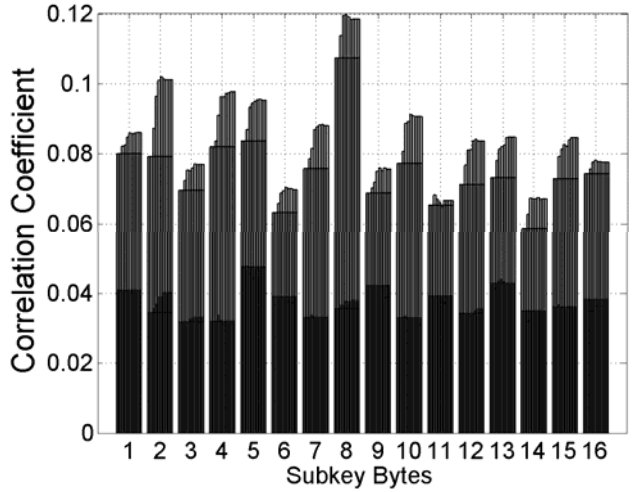
Correlation coefficients (fine grained)

AES-TBL

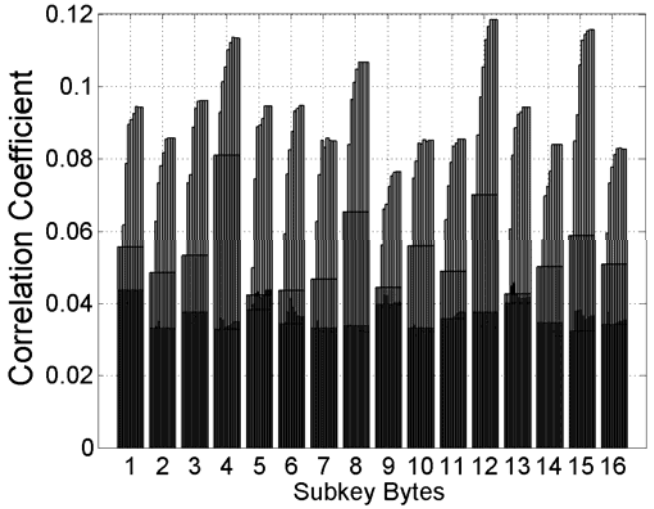


Each vertical bar consists of 10 vertical sub-bars, which correspond to the standard CPA and to the optimal CPA for u = 2,…,256, resp.

AES-PPRM3



AES-COMP



Design complexity and information gain

	no. of LUTs	Max. freq. (MHz)	max. timing (ns)
AES-TBL	1409	257.966	**4.558**
AES-PPRM3	2283	134.228	**8.138**
AES-COMP	2066	135.888	**11.274**

- *In our experiments the information gain of the optimal CPA over the standard CPA increased with the complexity of the Sbox design.*
- One might assume that high design complexity generally implies higher information gain. However, this requires further experiments with different designs.

Conclusion

- It has been well-known that the stochastic approach can be applied as an efficient attack tool and to obtain design information.
- In this talk we used the stochastic approach to derive the optimal CPA function to (arbitrary) given leakage distributions and for varying complexity of the leakage model.
- Experiments with three FPGA implementations of the AES cipher showed that the ‘information gain’ depends on the concrete implementation.

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